2nd Workshop on Proof Mining (WPM26)

University of Bath
September 7 – 9, 2026

Contents

https://proof-mining-workshop.github.io

Welcome Address

Proof theory started in the school of David Hilbert in Göttingen in the beginning of the 20th century with what is today known as Hilbert's program, a project in mathematical logic which aimed to show that the use of so-called ideal (i.e. non-constructive, set-theoretic or infinitary) principles in proofs of concrete so-called real statements could be (at least in principle) eliminated. In a modern view, this program is often subsumed under the goal of proving the consistency of powerful theories containing such ideal principles in certain finitistic theories. As is well-known, Gödel's second incompleteness theorem already rules out the provability of the consistency of many theories in themselves.

While Hilbert's program in this specific sense is therefore impossible to achieve globally, Georg Kreisel's work starting in the 1950's under the name of "unwinding proofs" marks the beginning of a seminal paradigm shift in proof theory, namely to use proof-theoretic devices developed in the course of Hilbert's program to extract computational information from specific given mathematical proofs, showing that Hilbert's program is actually largely achievable as far as most proofs from the literature are concerned. This field gained maturity in the 1990's through the work of Ulrich Kohlenbach, since then known as Proof Mining, and by now comprises over 280 works with many case studies situated within numerical mathematics and many branches of modern analysis.

Current research in this fast-growing area is concerned both with the development of new logical tools designed for facilitating a firm theoretical basis of this enterprise as well as with advancing the range of case studies into new territories relevant also for the mathematicians working in these fields in question.

This is the second meeting in a series of workshops dedicated to providing opportunities for people from the diverse areas connected to proof mining to meet, present work in progress and have an interdisciplinary exchange of ideas and knowledge. As such, the topics of interest in particular include logical aspects of proof interpretations as well as applied aspects from in particular nonlinear analysis and optimization.

The Organizers
Nicholas Pischke, Thomas Powell

Outline of the workshop

The Workshop on Proof Mining 2026 is hosted at the University of Bath from 7–9 September 2026, taking place in room 10W 2.47 on the main campus. Funding for the workshop was provided by the Association for Symbolic Logic, in the form of travel grants that were awarded to graduate students and recent PhDs to attend the workshop, the British Logic Colloquium, the Department of Computer Science of the University of Bath, and the London Mathematical Society under a Scheme 1 Conference Grant (Ref 12551).

The scientific programme comprises 23 talks from experts in both core proof mining and from important areas of application, and can be found at the back of this booklet, with abstracts for talks following this outline.

The list of participants in particular includes the following 26 people ordered alphabetically:

Miroslav Bačák	University of Leipzig (DE)
Radu Boţ	University of Vienna (AT)
Horatiu Cheval	University of Bucharest (RO)
Ernö Robert Csetnek	University of Vienna (AT)
Aris Daniilidis	TU Vienna (AT)
Luisa Després	TU Darmstadt (DE)
Jelena Diakonikolas	University of Wisconsin-Madison (US)
Nicoleta Dumitru	University of Bucharest (RO)
Fernando Ferreira	University of Lisbon (PT)
Ulrich Kohlenbach	TU Darmstadt (DE)
Angeliki Koutsoukou-Argyraki	Royal Holloway University of London (GB)
Ben Langton	University of Bath (GB)
Felipe Lara	Tarapaca University (CL)
Laurenţiu Leuştean	University of Bucharest (RO)
Genaro López-Acedo	University of Seville (ES)
D. Russell Luke	University of Göttingen (DE)
Morenikeji Neri	TU Darmstadt (DE)
Adriana Nicolae	Babeș-Bolyai University (RO)
Paulo Oliva	Queen Mary University of London (UK)
Pedro Pinto	University of Lisbon (PT)
Nicholas Pischke	University of Bath (UK)
Thomas Powell	University of Bath (UK)
Andrei Sipoş	University of Bucharest (RO)
Jacqueline Treusch	TU Darmstadt (DE)
Alex Wan	University of Bath (UK)
Jin Wei	TU Darmstadt (DE)

Abstracts of Talks

Quantitative results for primal-dual splitting methods for composite monotone inclusions

Ulrich Kohlenbach

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Recently primal-dual splitting algorithms for composite monotone inclusions of Tseng ([2]), Forward-Backward ([7]) and Douglas-Rachford ([1]) type have been quantitatively analyzed in [4],[3] and [5] resp. using proof-mining methods resulting in rates of metastability in the finite dimensional case as well as, under uniform monotonicity assumptions, full rates of convergence in general Hilbert spaces. As [2,7,1] each reduce the situation to usual forms of Tseng's algorithm, the Forward-Backward and the Douglas-Rachford algorithm with error terms, the logical analysis starts by giving quantitative accounts of the latter adapting the approach from [6] to the presence of error terms.

In this talk, we will compare the different type of algorithms from a logical point of view. More details on [1] will be provided in the talk by J. Treusch based on [5].

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Majorizability and uniformity principles

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The seminal work of Ulrich Kohlenbach initiated the exploration of the relationship between majorizability and functional interpretations. One of the facets of this exploration lead to the discovery of false uniformity principles and their elimination in proving certain mathematical statements. The bounded functional interpretation deepened our understanding of these principles and generalized them in the form of so-called collection and contra-collection principles. These principles are interpreted by the bounded functional interpretation. Recently, with Paulo Oliva, it was discovered that functional interpretations can be factored into a uniform part, where quantifications are computationally inert, and a computational part, where being of a certain type is imbued with computational information. When this information comes in the form of majorizability, we obtain a novel and more flexible view of the known uniformity principles. We discuss these matters, give some examples and make some observations and suggestions.

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Convexity Notions in Hadamard Spaces

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While convex optimization in geodesic spaces has traditionally relied on geodesically convex sets, horospherical convexity has recently emerged as a powerful tool for analyzing subgradient type algorithm convergence in Hadamard spaces. In this talk, we explore the interplay between these two notions of convexity and evaluate whether classic characterizations of closed, bounded convex sets from Hilbert spaces remain valid in this broader metric setting.

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Quantitative results on Halpern-type iterations

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Recently, in joint work with Leuştean [1], we analysed the quantitative asymptotic behaviour of the inexact generalized Halpern iteration for a single nonexpansive operator. In this talk, I present a work in progress that generalizes these results by replacing the single operator T with a countable family of nonexpansive operators $(T_n : X \to X)$ that satisfy a resolvent-like condition introduced in [2]. Specifically, I study the asymptotic behaviour of the following modified iteration:

$$x_0 \in X, \quad x_{n+1} = \delta_n f(x_n) + \alpha_n x_n + \beta_n T_n x_n + r_n,$$

where X is a normed space. Building on the proof mining techniques developed in [3, 4, 5], I extract uniform rates of asymptotic regularity and (T_n)-asymptotic regularity for this iteration. I will outline the main results obtained so far and provide concrete examples illustrating the application of this generalized framework.

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Quantitative Study of a Douglas-Rachford type Primal-Dual Algorithm

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Building up on the talk by U. Kohlenbach, this talk provides more information on the results of the quantitative analysis in [3] of a Douglas-Rachford type primal-dual algorithm due to Boţ and Hendrich in [1]. The goal is to give an overview of the extracted rates of convergence and metastability for the convergence results of this algorithm and to show how these rates relate to the ones that were extracted in [3] for the inexact Douglas-Rachford algorithm developed by Combettes in [2].

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Parameter-free approximation of fixed-point problems at the tractability frontier

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High-dimensional settings of fixed-point operator equations $T(x) = x$ arise across optimization, game theory, economics, and machine learning. When the operator T is contractive or nonexpansive, efficient algorithms with tight complexity guarantees are well established. For even mildly expansive operators, however, worst-case exponential lower bounds kick in. This raises a natural question: where exactly does the tractability frontier lie? In this talk, I will present recent results that identify new tractable regimes beyond nonexpansiveness. I will introduce the Adaptive Gradual Halpern Algorithm (AdaGHAL), a fully parameter-free method that recovers optimal oracle complexities for contractive and nonexpansive operators while extending guarantees to mildly expansive operators up to the known hardness boundary. I will then introduce the class of gradually expansive operators—permitting Lipschitz constants up to approximately $\sqrt{2}$—and show that AdaGHAL finds ϵ-approximate fixed points in $O(1/\epsilon)$ iterations in this regime. These results hold in general normed spaces, including infinite-dimensional Banach spaces. I will conclude with implications for root finding problems and weakly convex optimization, along with open questions.

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Self-consistent Estimators: Fixed Point Theory meets Statistics

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Many problems in both pure and applied mathematics can be formulated in terms of fixed point theory. Let U be a set and $Q\colon U \to U$ a mapping. Does Q have a fixed point? If so, is it unique? Beyond existence and uniqueness of a fixed point, a fundamental question is whether the iterations

$$u_{k+1} \coloneqq Q\left(u_k\right), \tag{1}$$

starting at some point $u_1 \in U$, converge to a fixed point. In this talk I will show an interesting application of this approach into statistics.

A very common problem in statistics is that data in time-to-event analyses are right-censored, that is, some observations are terminated before the event of interest occurs. However there is a well-developed mathematical framework that models such scenarios rigorously without loss of information. In particular, the Kaplan–Meier estimator [1] can be used for estimating the underlying probability distribution from right-censored data. Efron [2] identified this important estimator as a fixed point of a naturally arising mapping and called this property self-consistency; estimators that satisfy it are called self-consistent estimators. Efron also suggested that the iterative algorithm (1) might converge to a self-consistent estimator. Recent work [3] confirms that this is indeed the case. As a by-product one obtains the existence and uniqueness of a self-consistent estimator.

This talk is aimed at a general mathematical audience. After a brief introduction to time-to-event analysis with censored data, I will explain the concept of self-consistent estimators in detail and then discuss convergence of the iterations (1) in this context.

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Some results in stochastic analysis and optimization over metric spaces of nonpositive curvature

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I discuss some recent results in stochastic analysis on (separable) Hadamard spaces. The first cluster of these results is concerned with a stochastic variant of the proximal point algorithm for approximating zeros of the mean of a stochastically perturbed monotone vector field, where I discuss a strong convergence result, in mean and almost surely, under a suitable strong monotonicity assumption [6]. This result takes place over Hilbert-Hadamard spaces [5], that is separable complete geodesic metric spaces of nonpositive curvature where all tangent cones isometrically embed into a Hilbert space, and is moreover effective, that is furnished with an explicit rate of convergence. This lifts a result of Bianchi [3] for the first time to Hadamard manifolds, and provides the first quantitative information on this result already in Hilbert spaces.

In the special case of the proximal point algorithm for a random convex function, this restriction on the space can be lifted, now working over general (separable) Hadamard spaces, and one can produce explicit rates of convergence already under a significantly weaker and general stochastic regularity assumption [9]. Moreover, in this setting, one can establish the weak almost sure convergence of this method without any regularity assumptions over general (separable) Hadamard spaces [7], which solves a problem of Bačák [2].

The second cluster of results is concerned with strong laws of large numbers on Hadamard spaces [8]. Here, we extend Sturm's strong law of large numbers using inductive means on Hadamard spaces [13] from L^∞- to L^1-sequences of i.i.d. random variables by means of an effective non-asymptotic concentration inequality that moreover entails a distribution-uniform generalization of the strong law of large numbers, lifting a result of Chung [4] to Hadamard spaces.

We can then extend this strong law to integrable random monotone vector fields on separable Hilbert-Hadamard spaces. As a particular case, this transfers previous results of Salim [11] for the first time to Hadamard manifolds. We consider some special cases of our strong law for random monotone vector fields related to stochastic convex analysis and in that context in particular establish a result on the interchangeability of the subdifferential and the integral in the setting of separable Hilbert-Hadamard spaces where all tangent cones are actually full Hilbert spaces and where the integrand is an L^2-Lipschitz function, providing the first nonlinear version of (a particular case of) a previous result due to various authors, in particular Rockafellar and Wets [10]. This strong law for random monotone vector fields together with the interchangeability result can be used to establish a probabilistic Lie-Trotter-Kato formula for the gradient flow generated by an integral function over these spaces, also leveraging the previous Lie-Trotter-Kato formula established for gradient flows by Stojković [12] together with a result relating resolvent and gradient flow convergence established by Bačák [1].

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Strong convergence and fast residual decay for monotone operator flows via Tikhonov regularization

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In the framework of real Hilbert spaces, we investigate first-order dynamical systems governed by monotone and continuous operators. It has been established that for these systems, only the ergodic trajectory converges to a zero of the operator. However, trajectory convergence is assured for operators with the stronger property of cocoercivity. For this class of operators, the trajectory's velocity and the opertor values along the trajectory converge in norm to zero at a rate of $o(1/\sqrt{t})$ as $t \to +\infty$.

In this talk, we show that augmenting a monotone operator flow with a Tikhonov regularization term ensures not only strong convergence of the trajectory to the minimal-norm element of the zero set, but also enables the derivation of explicit convergence rates. In particular, we establish norm rates for the trajectory's velocity and for the residual of the operator along the trajectory, expressed in terms of the regularization function. In some particular cases, these rates can be as fast as $O(1/t)$ as $t \to +\infty$ [3]. In this way, we emphasize a surprising acceleration feature of the Tikhonov regularization. Additionally, we explore these properties for monotone operator flows that incorporate time rescaling and an anchor point. For a specific choice of the Tikhonov regularization function, these flows are closely linked to second-order dynamical systems with a vanishing damping term. The convergence and convergence rate results we achieve for these systems complement recent findings for the Fast Optimistic Gradient Descent Ascent (OGDA) dynamics [2].

When the monotone operator is given by the identity minus a nonexpansive operator, the monotone equation reduces to a fixed point problem. In this setting, an explicit discretization of the Tikhonov regularized system, enhanced with an anchor point, yields the Halpern fixed point iteration. We develop a general theoretical framework that provides convergence rates for the discrete velocity norm and the fixed point residual norm, serving as the discrete counterpart of the continuous time analysis [3]. For certain choices of regularization sequences, we obtain specific convergence rates that match those obtained in continuous time.

Finally, derive via an explicit discretization of the Tikhonov regularized monotone flow a novel Extra-Gradient method with anchor term governed by general parameters. We establish strong convergence to specific points within the solution set, as well as convergence rates expressed in terms of the regularization parameters. Notably, our approach recovers the fast residual decay rate $O(1/k)$ as $k \to +\infty$ for standard parameter choices [1].

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Stochastic First-Order Algorithms in Nonlinear Spaces

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We present our fixed point framework for the convergence analysis of stochastic algorithms in general metric spaces. Concrete motivations for our approach include molecular electronic structure reconstruction from X-FEL experiments, and simple statistical computations on spaces of phylogenetic trees. The theory is applied to fundamental, first-order algorithms, namely stochastic gradient descent and proximal splitting algorithms. Numerical results on idealized synthetic examples provide a proof of concept. Our framework provides a basis for evaluating and comparing different numerical strategies, as well as indicating the potential and limits for applications in nonlinear spaces.

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General frameworks for the convergence of stochastic algorithms

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Proof mining in probability and stochastic optimization has flourished over the last 2–3 years. Among many other achievements, it has resulted in a substantial collection of general quantitative convergence theorems for stochastic algorithms in the presence of quite broad assumptions. In the first part of this talk, I survey one such result, obtained in collaboration with Nicholas Pischke, on convergence under an abstract regularity condition [1]. In the second part, I discuss how this growing body of quantitative results might be exploited to semi-automate the extraction of computational content from convergence proofs in optimization, and present some ongoing joint work in this direction with Benjamin Langton.

References

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A Formalisation Library and a Prototype Logical System for Semi-automated Proof Extraction

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This talk consists of two main strands: first, the development of a formal library in Lean 4 for representing convergence proofs in optimisation; and second, the development of a prototypical domain-specific logical system aimed at enabling the semi-automated extraction of quantitative versions of formally encoded qualitative proofs. We use the formal library itself to facilitate this extraction in practice. This is work in progress, carried out jointly with Thomas Powell.

Quantitative limit theorems for generalized Pólya urns with applications to random tree models

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In this talk, we present recent advances in the quantitative study of stochastic processes within the proof mining program. Imagine an urn containing balls of finitely many possible colours. At each time step, we select a ball uniformly at random and return it. We then add or remove balls based solely on the colour of the ball we selected, with the rules governing the dynamics encoded in a matrix called the *replacement matrix* of the urn. A generalized Pólya urn is the stochastic process that tracks the state of the urn at each time step. We present new quantitative limit theorems on the asymptotic distribution of the respective ball colours in a generalized Pólya urn, which yield convergence rates. We note that our limit theorems are also qualitatively new. As an application, we recover the almost sure convergence of the distribution of the outdegrees in random recursive tree models, previously obtained by noneffective convergence results for Pólya urns, with our new limit theorems providing novel linear rates.

Subgradients in nonpositively-curved metric spaces

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In Euclidean spaces, subgradients naturally generalize classical derivatives and provide first-order optimality conditions for nondifferentiable convex functions. Extending this concept to nonlinear settings and developing corresponding optimization algorithms require subtle constructions. This talk presents several recently introduced notions of subgradients for functions on nonpositively-curved metric spaces and compares these approaches. Particular emphasis will be given to subgradients based on Busemann functions, algorithms relying on them, and some of their natural applications.

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Quantitative results on generalizations of the Krasnoselskii-Mann and Halpern iterations

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This talk presents recent applications of proof mining to the quantitative analysis of the asymptotic behaviour of generalizations of the Krasnoselskii-Mann and Halpern iterations recently studied by Kanzow and Shehu [5].

Let X be a normed space and $T : X \to X$ be a nonexpansive mapping with fixed points.

The *generalized Krasnoselskii-Mann-type iteration*, studied by Kanzow and Shehu [Section 3, 5] in Hilbert spaces and by Zhang, Guo, Wang [14] in classes of uniformly convex

Banach spaces, is defined by

$$x_0 \in X, \qquad x_{n+1} = \alpha_n x_n + \beta_n T x_n + r_n,$$

where $(r_n) \subseteq X$ and (α_n), $(\beta_n) \subseteq [0,1]$ satisfy $\alpha_n + \beta_n \leq 1$ for all $n \in \mathbb{N}$.

In [3], we compute uniform rates of asymptotic and T-asymptotic regularity for this iteration in uniformly convex Banach spaces. The proof mining techniques used to obtain these quantitative results were developed in [11,12] and are based on a combination of two logical metatheorems: one for classical systems [6] and one for semi-constructive systems [4].

Kanzow and Shehu also study in [Section 4, 5] the following generalization of the Halpern iteration:

$$x_0 \in X, \qquad x_{n+1} = \delta_n u + \alpha_n x_n + \beta_n T x_n + r_n,$$

where $(r_n) \subseteq X$ and (δ_n), (α_n), $(\beta_n) \subseteq [0,1]$ satisfy $\alpha_n + \beta_n + \delta_n \leq 1$ for all $n \in \mathbb{N}$. We refer to this scheme as the *inexact generalized Halpern iteration*.

We obtain in [1] quantitative and qualitative results on asymptotic and T-asymptotic regularity for a viscosity version of (x_n) by extending to this iteration the techniques developed in [7,10] for the Halpern iteration. For a particular choice of the parameter sequences, we get linear rates as an application of a lemma due to Sabach and Shtern [13]. Furthermore, in [2], we compute uniform rates of metastability of the inexact generalized Halpern iteration (x_n) by applying proof mining methods developed in [8,9] for convergence proofs that are based on Banach limits.

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Further quantitative moduli around uniform convexity

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Usually, the main obstacle to the activity of proof mining is that the arguments used in mathematical papers appeal to highly infinitary principles that may not admit a simple computational interpretation. In functional analysis, this would translate into the existence of 'ideal' points such as limits (be they limits superior or inferior). A close viewing of the proof may show that the use of these ideal principles is not really needed for getting to the conclusion of the theorem, at the expense of making the argument slightly messier.

We present some worked examples that illustrate this methodology. Firstly, we take a result of Prüß [2] which shows that uniform convexity of Banach spaces is equivalent to a different inequality governed by a modulus. The equivalence proof heavily uses limits superior. We explain how this may be rewritten into an argument that only uses inequalities of norms of ordinary points, and we show how it may be done quantitatively, deriving an expression of said modulus in terms of the modulus of uniform convexity.

Secondly, we move on to a class of 'nonlinear' spaces – metric spaces which are endowed with a generalization of the usual linear convexity structure – where we treat a property of the proximal mapping, a tool recently extended to this setting in [3]. Here, the main difficulty involves the elimination of tools used in uniformly Banach spaces which are specific to that setting: the dual and the subdifferential. We show how one may replace those by simple metric arguments and thus derive the sought modulus.

The results in this paper may be found in [4].

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Horofunctions and Metric Compactifications

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For a metric space (X, d), the horofunction extension is the closure of the normalized distance functions $x \mapsto d(x, z) - d(x_0, z)$ in the pointwise topology. This naturally gives a continuous injection of (X, d) to a compact space, but this construction might not be a topological compactification, since the original topology of X is not necessarily preserved. We present a necessary and sufficient metric criterion for it to be a topological compactification, with consequences for proper and ultra-metric spaces. In Banach spaces this becomes non-octahedrality of the norm, yielding renorming criteria and examples including reflexive spaces, infinite-dimensional L_1-spaces and ℓ_∞.

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Tikhonov regularization for monotone operators: dynamics

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We discuss the asymptotic behaviour of the trajectories of a second order dynamical system with Tikhonov regularization for solving a monotone equation with single valued, monotone and continuous operator acting on a real Hilbert space. We consider a vanishing damping which is correlated with the Tikhonov parameter and which is in line with recent developments in the literature on this topic. A correction term which involves the time derivative of the operator along the trajectory is also involved in the system and makes the link with Newton and Levenberg-Marquardt type methods. We obtain strong convergence of the trajectory to the minimal norm solution and fast convergence rates for the velocity and a quantity involving the operator along the trajectory. The rates are very close to the known fast convergence results for systems without Tikhonov regularization, the novelty with respect to this is that we also obtain strong convergence of the trajectory to the minimal norm solution. As an application we introduce a primal-dual dynamical system for solving linearly constrained convex optimization problems, where the strong convergence of the trajectories is highlighted together with fast convergence rates for the feasibility measure and function values.

Six Remarks on Functional Interpretations

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Functional interpretations are among the principal tools of the *proof mining* programme [14] – the systematic extraction of additional quantitative or qualitative information from mathematical proofs. One of the most successful is Kohlenbach's *monotone functional interpretation* [13], which combines Gödel's Dialectica interpretation [9] with Howard's majorizability relation [11]. Other prominent functional interpretations include Kleene realizability [12], Kreisel's modified realizability [15], the Diller-Nahm variant of the Dialectica interpretation [1], and the bounded functional interpretation [2,3].

In this talk I will revisit my own quest to understand the relationships between these different functional interpretations. I will present *six crucial observations* that have guided my work with collaborators and led to a number of developments in the field, including unifying functional interpretations [16], functional interpretations of intuitionistic linear logic [5,8,20] and classical linear logic [17,18,19], the hybrid functional interpretation [10,21], connections between different negative translations [6,7], and, more recently, the investigation of uniform functional interpretations [4,22].

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On Debreu-Koopmans Theorem for Additively Decomposed Quasiconvex Functions

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The Debreu-Koopmans theorem [1] shows that if a separable function

$$h(x_1, \ldots, x_n) = \sum_{i=1}^{n} h_i(x_i),$$

is quasiconvex, then *at most* one h_i might be nonconvex and, as a consequence, when all components h_i are quasiconvex, h is not quasiconvex and it is not known to which class of functions h belongs, giving rise to the Debreu-Koopmans problem.

In this talk, we introduce the class of (strongly) star quasiconvex functions [4], and we prove that h is (strongly) star quasiconvex if and only if all components h_i are (strongly) star quasiconvex [2] and, as a consequence, we solves the Debreu-Koopmans problem. This result has strong implications in economic theory, since formally bridges classical diversification theory with behavioral economics, as it implies that separable S–shaped (Sigmoid) value/utility functions from Prospect Theory [3] are star quasiconvex. Finally, and if the time allows us, we present potential applications in economics and further research directions.

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A Lean framework for bound extraction from formalized proofs

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We present some extensions of previous work on formalizing the metatheorems of proof mining in the Lean theorem prover, and creating bound extraction infrastructure based on them. Building upon this formalization, we construct a Lean embedded framework aimed at formalizing proofs in the WE-HA$^\omega$-based systems with abstract types that are used in proof mining. In order to address the practical challenges of spelling out formal proofs in such systems, the framework features a proof mode designed to mimic the higher-level manner in which native Lean proofs are written, and outputs derivations that can serve as inputs to the metatheorems. The framework is parametric in the abstract spaces, allowing common instances such as those for metric or normed spaces to be user-defined. We also discuss some case studies of theorems formalized in the framework, with extracted bounds. This is ongoing work.

On the proximal point algorithm for strongly quasiconvex pseudomonotone equilibrium problems in Hadamard spaces

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In [5], A. Iusem and F. Lara provide a convergence result for two proximal point type methods for finding equilibrium points of pseudomonotone and strongly quasiconvex bifunctions. In this talk, we discuss how their results can be lifted from the linear setting to Hadamard spaces in a quantitative way, following ideas in [7] and a quantitative analysis of Fejér monotonicity developed in [6]. Due to the elementary character of the proof, we will be able to discuss a strong convergence result under weakened assumptions than those featuring in [5] and provide effective rates of convergence for the iterates generated by such a procedure towards the solution. In this setting, we also briefly discuss [4] and its generalization to Hadamard manifolds. This talk is based on [2,3] and [1], the latter of which was written under the supervision of U. Kohlenbach and N. Pischke.

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Proof mining in nonlinear and stochastic settings

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In this talk, I will discuss some ongoing work [1,2] on quantitative aspects of convergence in nonlinear and stochastic settings.

A key feature of proof mining is its ability to unravel complex mathematical arguments, often revealing that general assumptions can be replaced by the specific instances actually used in a proof. This has proved particularly fruitful in transferring results from linear to nonlinear settings, where many of the combinatorial features of the original arguments can still be exploited. There are however limitations and the absence of a linear structure may prevent a direct generalization of such results. In this regard, a finitistic (i.e. quantitative) perspective can provide a way around these obstacles and has led to new results in settings where conventional approaches appear to be insufficient; see, for example, [3,4].

I will first discuss recent work [1] on the convergence of a generalized viscosity implicit rule in Hadamard spaces. The analysis is carried out from a finitistic perspective and illustrates how quantitative methods can be used to overcome difficulties arising from the lack of linear structure.

A more recent direction in proof mining, spearheaded in particular by the group in Bath [5,6], concerns applications to probability theory. The quantitative analysis of stochastic methods has proved especially fruitful, leading to both new convergence results and novel quantitative information. In the second part of the talk, I will present ongoing joint work with Nicholas Pischke [2] on the extraction of a rate of metastable uniform convergence for a stochastic version of the Halpern iteration in the general metric setting of Hadamard spaces (see also [7]).

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7-9 September 2026
University of Bath

II. WORKSHOP ON PROOF MINING

Participants include

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Website: https://proof-mining-workshop.github.io

Program

	MONDAY	TUESDAY	WEDNESDAY
09:15 - 09:30	Opening Address		
09:30 - 10:15	**Kohlenbach**	**Boţ**	**Daniilidis**
10:15 - 11:00	**Ferreira**	**Luke**	**Csetnek**
11:00 - 11:30	COFFEE BREAK		
11:30 - 12:15	**López Acedo**	**Powell**	**Oliva**
12:15 - 12:40	**Dumitru**	**Langton**	**Lara**
12:40 - 13:05	**Treusch**	**Neri**	
13:05 - 14:30	LUNCH		
14:30 - 15:15	**Diakonikolas**	**Nicolae**	**Cheval**
15:15 - 16:00	**Bačák**	**Leuştean**	**Després**
16:00 - 16:30	COFFEE BREAK		
16:30 - 17:15	**Pischke**	**Sipoş**	**Pinto**
17:15 - 17:30			Closing Address